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General purpose technologies 'Engines of growth'?

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Abstract

Whole eras of technical progress and growth appear to be driven by a few 'General Purpose Technologies' (GPT's), such as the steam engine, the electric motor, and semiconductors. GPT's are characterized by pervasiveness, inherent potential for technical improvements, and 'innovational complementarities', giving rise to increasing returns-to-scale. However, a decentralized economy will have difficulty in fully exploiting the growth opportunities of GPT's: arms-length market transactions between the GPT and its users may result in 'too little, too late' innovation. Likewise, difficulties in forecasting the technological developments of the other side can lower the rate of technical advance of all sectors.

Key words: Technical change; Growth; Social returns; Coordination
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1. Introduction

Economists have known for a long time that technical change is the single most important force driving the secular process of growth (Abramovitz, 1956; Solow, 1957). Yet, relatively little progress has been made in accounting for the

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'residual' of aggregate production functions,[1] largely because economic theory tends to treat all forms of technical change in the same, diffuse manner. In fact, we can hardly distinguish in our models between a momentous invention such as the transistor and the development of yet another electronic gadget.

By contrast, economic historians emphasize the role played by key technologies in the process of growth, such as the steam engine, the factory system, electricity, and semiconductors (Landes, 1969; Rosenberg, 1982). Anecdotal evidence aside, are there such things as 'technological prime movers'? Could it be that a handful of technologies had a dramatic impact on growth over extended periods of time? What is there in the nature of the steam engine, the electric motor, or the silicon wafer, that make them prime 'suspects' of having played such a role?

In this paper we attempt to forge a link between the economic incentives for developing specific technologies and the process of growth. The central notion is that, at any point of time, there are a handful of 'general purpose technologies' (GPT's) characterized by the potential for pervasive use in a wide range of sectors and by their technological dynamism. As a GPT evolves and advances it spreads throughout the economy, bringing about and fostering generalized productivity gains.

Most GPT's play the role of 'enabling technologies', opening up new opportunities rather than offering complete, final solutions. For example, the productivity gains associated with the introduction of electric motors in manufacturing were not limited to a reduction in energy costs. The new energy source fostered the more efficient design of factories, taking advantage of the newfound flexibility of electric power. Similarly, the users of micro-electronics are among the most innovative industries of modern economies, and they benefit from the surging power of silicon by wrapping around the integrated circuits their own technical advances. This phenomenon involves what we call 'innovational complementarities' (IC), that is, the productivity of R&D in a downstream sector increases as a consequence of innovation in the GPT technology.[2] These complementarities magnify the effects of innovation in the GPT, and help propagate them throughout the economy.

Like other increasing returns-to-scale phenomena, IC create both opportunities and problems for economic growth through technical advance. Development of GPT-using applications in a wide variety of sectors raises the return to new advances in the GPT. Advances in GPT technology lead to new opportunities for applications. Such positive feedbacks can reinforce rapid technical

[1] See, however, the series of papers in Parts II and IV of Griliches (1988).

[2] In defining innovational complementarities and understanding their role, we were strongly influenced by Rosenberg's insightful 1979 essay, 'Technological Interdependence in the American Economy', reproduced in Rosenberg (1982). The formal analysis is close in spirit to that of Milgrom et al. (1991).

progress and economic growth. The problem is that these complementary innovative activities are widely dispersed throughout the economy, making it very difficult to coordinate and provide adequate innovation incentives to the GPT and application sectors.

These difficulties are hardly surprising, considering that uncertainty and asymmetric information, which make coordination difficult, are essential features of the process of new knowledge creation (Arrow, 1962). Moreover, time gaps and sequentiality are an inherent feature of technological development, particularly in the context of GPT's (e.g., the transistor could not come before electricity, nor could interferon before DNA). Therefore, coordination in this context would require aligning the incentives of agents located far from each other along the time and the technology dimensions. Since GPT's are connected by definition to wide segments of the economy, coordination failures of this nature may have far reaching consequences for growth.

A great deal of theoretical work has been done in recent years on the role of increasing returns in endogenous growth, going back to Romer's (1986) contribution. However, many of these models regard the economy as 'flat', in that they do not allow for explicit interactions between different sectors.[3] The *locus* of technical change does not matter much in those models, and hence there is little room to discuss explicitly the industrial organization of inventing sectors. Closely related, technical change is often assumed to be all-pervasive, that is, to occur with similar intensity everywhere throughout the economy. Clearly, one could not build a theory of growth that depends upon the details of bilateral market relations, when those details could refer to any or all of the myriad markets that make up the economy. By contrast, we identify here a particular sector (the GPT prevalent in each 'era') that we regard as critical in fostering technical advance in a wide range of user industries, and presumably in 'driving' the growth of the whole economy. The price that we pay, though, for the sharp focus is that the analysis is partial equilibrium, and hence the implications for aggregate growth stem just from the supply side, and abstract from general equilibrium type of feedbacks.[4]

We organize the analysis in order to draw two sets of implications out of a simple model of decentralized technological progress. In Sections 2 and 3 we consider the implications of generality of purpose and innovational complementarities for the economy-wide incentive to innovate. These sections emphasize the vertical relations between the procedures in GPT and application sectors, and the dual appropriability problem that arises in that context. Section

[3] For a notable exception see Grossman and Helpman (1991), particularly their models of the product cycle.

[4] Murphy et al. (1989) show that partial equilibrium implications are robust to general equilibrium considerations in a model with aggregate logic much like ours, but different microfoundations.

4 turns to a set of dynamic issues: the role of bilateral inducement of technical progress over time, the difficulties of the GPT and AS sectors in forecasting each others' rate of progress, and the consequent 'too little, too late' decisions that slow the arrival of social gains from a GPT. Section 6 concludes with directions for further research.

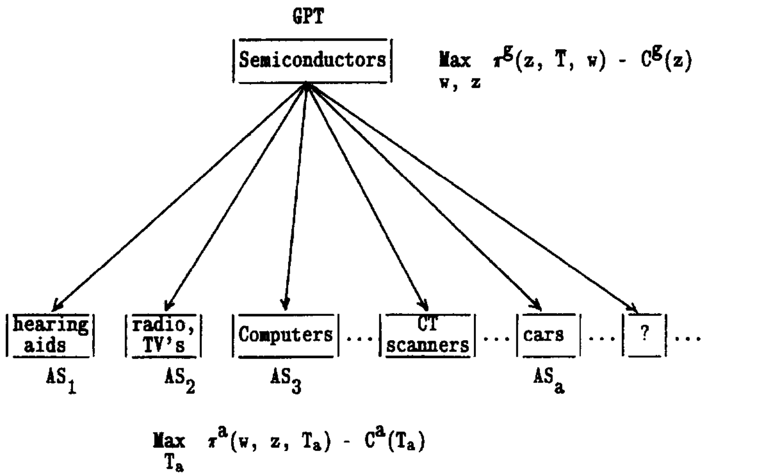
One of Zvi Griliches's earliest contributions was the study of hybrid corn, a technology that can surely be regarded as a GPT in the context of agriculture. Indeed, that is how Griliches himself (1957, p. 501) perceived it: 'Hybrid corn was the invention of a method of inventing, a method of breeding superior corn for specific localities.' Many of the themes of our analysis are also familiar from Griliche's work: the private incentive to adopt new technologies (Griliches, 1957), the return to innovations at the firm level (Griliches, 1958, 1984), and the causes and consequences of returns-to-scale (Griliches, 1971). This paper attempts to integrate these themes in the hope of illuminating some broader phenomena.

2. Incentives to innovate in the GPT and application sectors

Several common themes emerge from surveying past and present GPT's.[5] A generic function (or 'general concept') such as 'continuous rotary motion' for the steam engine, or 'transistorized binary logic' for the integrated circuit, can be applied in many sectors. Yet advancing the performance of objects embodying these functions and making them economically viable pose great challenges. Thus, cheaper steam power called for mechanically better engines using improved materials, and a superior understanding of thermal efficiency. More advanced integrated circuits have their own complex logic, but also call for advances in photolithography and other manufacturing processes. Finally, making the general concept work in any specific situation requires further complementary innovation, and often a great deal of ingenuity. Who knew that continuous rotary motion could make sewing cheaper, or that carburetion in an automobile engine and addressing envelopes were binary logic activities? These observations about technology inform and drive the forthcoming analysis.

Our model is of a stylized set of related industries with highly decentralized technical progress, centered around the GPT. To fix ideas, think of this sector as semiconductors. The level of technology in that sector, called z, appears to users in the application sectors as quality attributes. In semiconductors these are the speed, complexity, functionality, size, power consumption, reliability, etc. of integrated circuits. While almost all interesting real-world examples have

[5] We do not attempt any serious review of the relevant technological facts here. Section 2 of Bresnahan and Trajtenberg (1992) has more detail. See also Mokyr (1990) and David (1990).



Notation:

GPT:	General Purpose Technology
AS_a:	Application Sector a
z:	"Quality" of the GPT
w:	Market price of the GPT
c:	Marginal cost of GPT
T_a:	Technological level (or "performance") of AS_a (T: vector of T_a's)
π^g, π^a:	Gross rents of the ith sector, i: GPT, AS_a
C^i:	R&D costs of the ith sector, i: GPT, AS_a

Fig. 1. The framework of analysis.

multidimensional z, we formulate the model in terms of a scalar z.[6] Finally, the economic return to improved technology in the GPT comes by selling a good embodying the technology at price w, in markets where the GPT firm(s) exercise some degree of monopoly power. The economic incentives for innovation in the GPT depend on the prevailing market structure and appropriability, as well as on the demand function in the applications sectors, which determines GPT revenue as a function of w and z.

In this framework an applications sector (AS) is an actual or potential user of the GPT as an input; each AS engages in its own innovative activity, leading to a level of own technology T_a. Fig. 1 shows some the AS's using semiconductors:

[6] Think of it as the density with which transistors can be packed on a chip, the fundamental level of semiconductor technology which permits advances along most of the quality axes.

early transistors were incorporated in hearing aids, shortly after in radios, computers, and then television sets. The development of the integrated circuit permitted applications in many entirely new products (e.g., CT scanners, camcorders). A particularly important subclass of integrated circuits is microprocessors, which permitted the creation of 'smart devices' (personal computers, laser printers, automobile engine control systems). In parallel to the appearance of new applications, the GPT fosters continued change in existing sectors such as military or civilian aircraft. More generally, adapting and adopting the GPT for different sectors is itself an innovative activity. For GPT's as pervasive as semiconductors, innovation in the different AS's can be very diverse. What is shared among applications sectors is the level of GPT technology and not necessarily any other economic feature.[7]

We begin by modelling the incentives to innovate facing the GPT and AS's. The key technical assumptions are generality of purpose and innovational complementarities.[8] These translate, in a world of imperfect appropriability, into two distinct externalities: the 'vertical' externality between the GPT and each application sector, and the 'horizontal' one across application sectors. We then examine the welfare consequences of these externalities in the context of a simple one-step innovation game.

2.1. Modelling the application sectors

Each application sector (indexed by a) determines the level of its own technology, $T_a \geqslant 0$, and its demand for the GPT good, X^a. The objective function which the single AS acts *as if* it maximizes is

$$\max_{T_a} \Pi^a(w, z, T_a) - C^a(T_a) \equiv V^a(w, z), \tag{1}$$

where $C^a(\cdot)$ are invention costs and Π^a stands for the gross private returns to technical advance in the AS. We assume that if $V^a(w, z) < 0$ the sector takes

[7] Not all historically important GPT's have this industrial organization. Many developments in early steam engine technology, for example, took place inside using industries such as mining and transportation. Our modelling strategy is to associate each technology with a separate economic agent, so as to illuminate the incentive to innovate for each. We then revisit the question of how these agents might be organized, by firms, contracts and markets.

[8] We omit here two additional forces that are thought to play a similar role: Technological interrelatedess and diffusion in conjunction with learning-by-doing. The first means that there is 'learning by inventing'. The invention of particular subtechnology in the context of a GPT lowers the cost of inventing the next one, which, in turn, contributes to span other subtechnologies further down the line. The second is more conventional: As the number of downstream sectors using the GPT increases, the costs of producing the generalized input go down because of 'learning-by-doing', thus contributing to a self-sustained process of economy-wide growth.

some opportunity action with value normalized to zero.[9] The definitions of z, T, and w imply that $\Pi^a_Z > 0$, $\Pi^a_T > 0$, and $\Pi^a_W < 0$.

It is standard in models of vertical integration to treat the downstream sector as a single entity, and hence to refer to Π^a as 'the' payoff to the AS, without distinguishing between buyers and sellers within the sector; see, for example, Hart (1988) and Bolton and Whinston (1993). In general, though we do not expect market arrangements within any given AS to result in optimal innovation incentives. Thus, we provide in Section 2.2 three examples of the economic underpinnings of Π^a; in all of them Π^a moves together with social surplus in response to changes in z, w, or T_a, but only in one of the examples Π^a is identical to social surplus.

As usual in these types of models, we assume $C^a_T > 0$ and $C^a_{TT} > 0$. Most importantly, we assume the presence of 'innovational complementarities' (IC),

$$\Pi^a_{zT} \geqslant 0. \tag{2}$$

In words, the marginal value of enhancing the AS's own technology rises with the quality of the GPT. The solution to (1) leads to the technology investment function of the AS,

$$T_a = R^a(w, z). \tag{3}$$

It is immediate from the assumption of IC that $R^a(\cdot)$ is upward-sloping in z: technological improvements in the GPT *induce* complementary innovation in the AS. Relying on Shephard's lemma, the demand of the AS for the GPT input is given by

$$X^a(w, z, T_a) = -\Pi^a_w(w, z, T_a), \tag{4}$$

and hence the expenditure on purchasing the GPT input is $E^a = w\,X^a(w, z, T_a)$, which is obviously the revenue function of the GPT. Since the GPT sector will be assumed to exercise monopoly power, it is convenient to state some of the assumptions in terms of the 'inverse' marginal revenue function, $\widetilde{MR} \equiv \partial E^a/\partial w = (X^a + wX^a_w)$.

We assume that demand is downward-sloping ($X_w < 0$) and that $X_{T_a} > 0$, $X_Z > 0$, that is, that superior technology is demand-enhancing. We also assume that $X_{WZ} \leqslant 0$, which implies that demand does not become steeper as it shifts up following a quality upgrade in the GPT. This ensures that a GPT monopolist cannot appropriate more than the incremental surplus stemming from an

[9] If the GPT is critical for the very existence of the AS (e.g., semiconductors in microcomputers), then the value of the 'opportunity action' is identically zero; if the GPT is a noncritical enhancement (e.g., semiconductors in motor vehicle engine control), the opportunity action would be the use of an alternative technology.

increase in z, Π^a_z; thus, the GPT producer will underprovide z (as in Spence, 1975).

We make the following assumptions about the derivatives of $\widetilde{MR}$ (mirroring those about demand):

$$\widetilde{MR}_w = -(2\Pi^a_{ww} + w\Pi^a_{www}) < 0,$$

$$\widetilde{MR}_{T_a} = -(\Pi^a_{wT_a} + w\Pi^a_{wwT_a}) > 0,$$

$$\widetilde{MR}_z = -(\Pi^a_{wz} + w\Pi^a_{wwz}) > 0,$$

that is, z and T_a are assumed to shift marginal revenue in the same direction as the shift in demand; these, together with the (conventional) assumption that $\widetilde{MR}_w < 0$, ensure that the return to the GPT producer from investing in quality upgrades increases with T_a (see Appendix 1).

So far we have discussed the behavior of a single application sector, but the very concept of a GPT implies the existence of multiple AS's. For simplicity, assume that for all $\{z, w\}$ the ranking of AS's according to the maximized value of their payoff, $V^a(w, z)$, is the same. In that case the marginal AS is uniquely determined by the smallest positive $V^a(w, z)$. Let $A(w, z)$ be the set of sectors that find it profitable to use the GPT; clearly, $A(\cdot)$ will include more sectors the larger z is, and fewer sectors the higher is w. Thus, holding prices fixed, higher z's induce higher level of T_a in each active AS, and cause an expansion of the set of AS's by making it profitable for extramarginal sectors to adopt the GPT.

2.2. *Examples of the economic underpinnings of Π^a*

We provide here three alternative interpretations of $\Pi^a(\cdot)$, all having distinct buyers and sellers within each AS: sellers purchase the GPT good, combine it with their own technology, and sell their output to buyers. The examples differ in the assumptions regarding the industrial organization of the AS, and therefore in the relationship between Π^a and total surplus.

Consider first the case where there is perfect, costless contracting between buyers and sellers in the AS, and hence assume that they are able to design the cost-minimizing industry structure (just as they would if they were to vertically integrate). That is, they sign contracts which just cover $C^a(T_a)$ without affecting any allocational decision at the margin. Let the payoff to buyers be equal to the consumer surplus, $CS(P_a, z, T_a)$, where P_a is the price of the AS good;[10] thus, quantity demanded equals $-CS_P(P_a, z, T_a)$. On the supply side, denote by

[10] In our semiconductor example, the surplus could result from using personal computers based on microprocessors of quality z, and embodying computer architectures and other components of quality T.

$\gamma(z, T_a)$ the unit cost function, and assume that one unit of the AS good uses one unit of the GPT input (as one microcomputer uses one microprocessor). Then,

$$\Pi^a(w, z, T_a) \equiv \max_{P_a} CS(P_a, z, T_a) - [\gamma(z, T_a) + w][-CS_{P_a}(P_a, z, T_a)], \tag{5}$$

that is, Π^a maximizes consumers' surplus minus costs, including the costs of buying the GPT good. In other words, the AS acts in this case *as if* it maximizes total surplus of the sector.[11]

Eq. (5) shows also that one of the likely sources of innovational complementarities (IC) is $CS_{zT} > 0$. For example, final demanders of hearing aids are better off only if the quality of the transistor z is designed into the listening device via T_a. It is the quality of their improved hearing, which relies on the two technologies, that drives the IC in Π^a. Notice also that $P_a = \gamma(z, T_a)$, and hence $X^a(w, z, T_a) = -CS_{P_a}[\gamma(z, T_a), z, T_a]$. This may provide a rationale for the assumption that $X_{zT} > 0$, which we shall rely upon later on: the assumption holds if the IC arise indeed from $CS_{zT} > 0$.

The first-best contracts of the previous example may be difficult to negotiate or enforce. As a second example, suppose instead that price-taking buyers face a monopoly seller in the AS. Then, using the same notation,

$$\Pi^a(w, z, T_a) \equiv \max_{P_a} [P_a - \gamma(z, T_a) - w][-CS_{P_a}(P_a, z, T_a)].$$

Thus, Π^a is in this case the part of the surplus that is captured by the monopoly seller, and not *total* surplus as in the first example.[12]

As a third and final example, suppose that the *sellers* in the AS are price takers; once again Π^a would be producer's surplus in the AS, which will be zero if the firms in the AS are all identical with flat marginal costs. Clearly, the gap between Π^a and social welfare will be larger than in the monopoly case.

These examples make it clear that in a wide range of cases the payoff function governing the behavior of the AS is highly correlated with total sector surplus, though not necessarily identical to it. In any event, our analysis focuses on the efficiency of the productive sector of the economy, and abstracts from spillouts to the consumers of the AS's.

[11] Note that this formulation takes T_a as given and assumes that $C^a(T_a)$ is financed according to the terms of the contract struck between buyers and sellers.

[12] Notice that in this case the demand for the GPT input is $X^a(w, z, T_a) = X^a[\gamma(z, T_a)/(1 + \eta^{-1}), z, T_a]$, where η is the elasticity of demand for the AS goods.

2.3. *Incentives for innovation in the GPT sector*

We assume that the GPT sector sells an undifferentiated product and that it exercises monopoly power in setting its price w.[13] Thus, the restricted profit function is

$$\Pi^g(z, \bar{T}, c) \equiv \max_w (w - c) \sum_{a \in A} X^a(w, z, T_a), \tag{6}$$

where c is the constant marginal cost of producing the good embodying the GPT, $\bar{T}$ is the *vector* of technology levels of the AS's, and $A = A(w, z)$. The innovating behavior of the sector is characterized by (for $z \geqslant 0$)

$$\max_z \Pi^g(z, \bar{T}, c) - C^g(z), \tag{7}$$

where $C^g(z)$ stands for the cost-of-innovating function, exhibiting $C_z > 0$ and $C_{zz} > 0$. The solution to (7) gives us the reaction function

$$z = R^g(\bar{T}, c),$$

which will be upward sloping in $\bar{T}$ (the proof is in Appendix 1): we have assumed that higher T_a's shift demand *and* marginal revenue up, hence the private return to investment in z increases with T_a.

This is then the second half of a dual inducement mechanism: an improvement in the technology of any AS increases the incentives for the GPT to upgrade its technology, just as a higher z prompts the AS's to invest in higher T_a's. The technology levels of the AS's and of the GPT, $\{\bar{T}, z\}$, can be thus characterized as 'strategic complements' (Bulow et al., 1985).

2.4. *Equilibrium in the market for the GPT and the social optimum*

Assuming that the GPT and the AS's engage in arms-length market transactions (and hence ruling out technological contracting or other forms of cooperative solutions), we can easily characterize the (Nash) equilibrium as follows (we rely here on Milgrom and Roberts, 1990): $\{\bar{T}^o, z^o\}$ is an equilibrium if

$$T_a^o = R^a(z^o), \quad \forall a, \quad \text{and} \quad z^o = R^g(\bar{T}^o),$$

where for some AS's it may be that $T_a^o = 0$. The multiplicity of potential participant AS's will tend to induce multiple equilibria. There is always a 'low' equilibrium (i.e., $\{0, 0\}$) and, if the reaction functions are concave, at least one

[13] We abstract from the internal organization of the GPT 'sector', and treat it as a monopoly; however, the analysis below holds for pricing rules other than monopoly pricing.

interior equilibrium. Different numbers of participating AS's may support other interior equilibria as well. Moreover, one can always define *constrained* equilibria, one for each subset $A \subseteq \hat{A}$, where $\hat{A}$ is the superset of all possible AS's. The plausibility of alternative equilibria is interesting in itself; however, here we are interested primarily in analyzing the efficiency of different vertical arrangements *vis-a-vis* the social optimum. Thus, for comparison purposes we choose the 'best' decentralized equilibrium, that is, the one exhibiting the largest A, denoted by A^o, which will be associated also with the largest z^o and $\bar{T}^o$.

Now to the social optimum. First we impose marginal cost pricing ($w = c$), which implies $\Pi^g = 0$. For any $A \subseteq \hat{A}$ the social planner's problem is

$$\max_{z,\,T_a} \left[\sum_{a \in A} \Pi^a(c, z, T_a) - \sum_{a \in A} C^a(T_a) - C^g(z) \right] \equiv S(A). \tag{8}$$

Denoted by $\{T^*, z^*\}$ the arguments that fulfill (8); likewise,

$$A^* = \underset{A}{\operatorname{argmax}}\, S(A).$$

Proposition 1.[14] *The social optimum entails higher technological levels than the decentralized equilibrium, i.e.,* $z^* > z^o$, $T_a^* > T_a^o$, $\forall a$, *and* $A^o \subseteq A^*$.

The reason for the divergence between the social optimum and the decentralized Nash equilibrium lies in the complementarities between the two inventive activities and the positive feedbacks that are generated. Consider the following thought experiment: starting from the social optimum $\{z^*, \bar{T}^*\}$ and reasoning 'backwards', each player would want to innovate *less*: lowering z lowers each T_a which, in turn, means less commercial opportunity for the GPT sector, and hence a lower z. Moreover, a lower z means lower Π^a's, resulting in reductions in the size of $A(w, z)$ as some AS's payoffs to utilizing the GPT become negative. This means that the market for the GPT shrinks, prompting a further cutback in z, and hence in the T_a of those applications sectors that remain active.

It is important to note that the assumption of monopoly pricing by the GPT is *not* the villain, as can be seen by considering alternative pricing mechanism. First, pick a pricing rule that gives the AS's the right incentives to innovate: the only such rule is $w = c$, which leads to no appropriability and thus no innovation in the GPT. Second, attempt to pick a pricing rule that gives the GPT the social rate of return to innovation. Clearly, a single $w(\cdot)$ would not suffice, only the perfectly price-discriminating GPT monopolist would earn the social return.

[14] The proof closely follows Cooper and John (1988) and Milgrom and Shannon (1992), and hence we omit it. We note only that it relies on $R^a(z)$ and $R^g(\bar{T})$ being upward-sloping and on the assumption that $X_{zw} \leqslant 0$ (made in Section 2.1), which implies $\Pi_z^g(\cdot) \leqslant \Sigma_{a \in A} \Pi_z^a(\cdot)$.

But price discrimination would leave zero returns to technical advance in the AS's. A fully specified technology contract might solve the problem if it is binding (a big 'if'), but that just underlies the point made here: any arms-length market mechanism under innovational complementarities necessarily entails private returns that fall short of social returns for either upstream or downstream innovations, under *all* plausible pricing rules.

3. Two positive externalities

The feedback mechanism leading to social rates of return greater than private returns reflects two fundamental externalities. The first is vertical, linking the payoffs of the inventors of the two complementary assets, and follows from innovational complementarities. The second is horizontal, linking the interests of players in different application sectors, and is an immediate consequence of generality of purpose.

The vertical externality is closely related to the familiar problem of appropriability, except that here it runs both ways, and hence corresponds to a *bilateral* moral hazard problem (Holmstrom, 1982; Tirole, 1988). Firms in any AS and GPT sector have linked payoffs; the upstream firm would innovate only if there is a mechanism (involving $w > c$) that allows it to appropriate some of the social returns. The trouble is that, for any $w > c$, the private incentive for downstream innovation is too low. Thus, any feasible pricing rule implies that neither side will have sufficient incentives to innovate.

Recently, several scholars as well as industrial advocates have suggested broad-based changes in government policy to increase appropriability in sectors that would qualify as GPT's (primarily semiconductors). Typically, these policy initiatives concern intellectual property protection, limits on foreign competition, and the relaxation of antitrust standards for these sectors. Our analyses suggests that policy measures of this nature cannnot sensibly be evaluated in isolation. To be sure, such measures would improve the incentive to innovate in the GPT sector, but they might also lower the returns to complementary investments made by users of the GPT throughout the economy.

The second externality stems from the generality of purpose of the GPT. From the vantage point of the GPT, the AS's represent commercial opportunity; thus, the more AS's there are and the larger their demands, the higher will be the level of investment in the GPT technology. From the point of view of the AS's, expansions in the set A, enhancements to T_a, and increases in the willingness to pay for the GPT by any AS's makes all other AS's better off by raising z. Yet, in equilibrium, each AS finds itself with too few 'sister sectors', each innovating too little.[15] The point is that z acts like a public good while R^g is the fixed cost needed to produce that good. However, in contrast to the traditional analysis of public goods, attempts to cover such costs with transfer prices impose a tax that discourages innovation.

The horizontal externality illuminates policy issues in the economics of technology connected with the role of large, predictable demanders. It is often claimed that the procurement policy of the U.S. Department of Defense (DOD) and NASA 'built' the microelectronics-based portion of the electronics industry in the U.S. during the fifties and sixties. Obviously, the presence of a large demander changes the conditions of supply, and this may benefit other demanders. However, NASA and the DOD also had a high willingness to pay for components embodying z well outside current technical capabilities, and were willing to shoulder much of the risk through procurement assurances. In so doing NASA and the DOD may have indeed set in motion (and sustained for a while) the virtuous cycle mediated by the horizontal externality.

However, it is only a coincidence that the horizontal spillouts came from the demand activities of government agencies. In the same technology, large private demanders such as the Bell System and IBM contributed directly to the development of fundamental advances in microelectronics. Earlier GPT's displayed similar patterns, as for example in Rosenberg's (1982) description of the importance of improvements in the quality of materials for 19th century U.S. growth. Much of the private return to improvements in material sciences (and engineering) came from a few key *private* sectors, notably transportation. The need to build steel rails for the railroad and to contain steam in both railroads and steamships provided a type of demand parallel to that of the government body noted above. Focused on improvements in inputs that press the technical envelope, having high willingness to pay because of their own technological dynamism, such demanders provide substantial horizontal spillouts to the extent that the technical progress that they induce is generally useful.

These examples seem to suggest that 'triggers' often take the form of exogenous forces that shift the rate of return to GPT technology. Thus in the 19th century, the importance of certain sectors (e.g., transportation), driven by the economic development of the country, may have been the key. In the post Wold War II era, the onset of the Cold War resulted in a government procurement policy which may have played a similar role. In each case, the positive feedback aspects of GPT and AS developments then took over, generating very large external effects, and unleashing a process of technical change and growth that played out for decades.

[15] Note that this issue arises above and beyond the multiple equilibrium problem, since we have assumed that the 'best' Nash equilibrium is the one that holds.

3.1. Externalities and technological contracting

Clearly, the vertical and horizontal externalities offer strong motives for breaking away from the limitations of arms-length market transactions, by increasing the degree of cooperation and explicit contracting between AS's and the GPT and between the AS's themselves. To illustrate, consider the case where any two sectors can form a binding technology contract, be it the GPT sector and an AS or a pair of AS's. In the former case they will pick z and T_a to maximize $(\Pi^a + \Pi^g)$; in the latter, they will pick the two T_a's to maximize the sum of the two AS's payoffs. The result of either contract will be that z and T_a will be larger for *all* application sectors. Payoffs will be larger for the GPT sector and for all AS's not party to the contract as well. Note, however, that the activity of forming binding technology contracts is subject to the same externality as the provision of technology itself. Just as every AS would like to see other AS's advancing their own technology, so too each sector would like to see others making technology-development contracts with the GPT. Clearly, lack of enforceability as well as imperfect technology forecasting limit the practical importance of contracting.

Recent events in the computer and telecommunications markets show how these considerations work in the real world. For many years, coordination between GPT-related sectors and their AS's was made simpler by the presence of dominant firms such as IBM and AT&T. These firms took a leading role not only in the development of the GPT, but also in the encouragement of complementary innovations in specific directions. This ability to commit to specific technological trajectories and therefore to direct the overall innovation cluster was labelled 'credibility' by those AS who benefited from the tacit coordination, whereas those that did not saw it just as the exercise of plain market power.

Over time, technological and regulatory forces have significantly reduced the leading role of these dominant firms. There is no longer a single actor who can direct technical progress, but instead there are a few innovators of both complementary and competing technologies that influence the gradient of advance in GPT-related industries. In parallel, a wide range of weaker mechanisms have emerged for coordinating and directing technical progress. 'Strategic alliances', participation in formal standards-setting processes, consortia, software 'missionaries', and the systematic manipulation of the trade press have all emerged as standard management tools in micro-electronics-based industries. These mechanisms permit both revelation of the likely direction of technical advance within particular technologies and the encouragement of complementary innovations. Yet they probably fall short of offering the means to internalize the bulk of the externalities discussed above.

4. The dynamics of general purpose technologies

In previous sections we assumed a one-shot game, allowing us to discuss the two main externalities associated with GPT's. We turn now to dynamic aspects of the performance of GPT's, such as the role of informational flows between sectors and their implications for growth. A suitable framework to model the way by which the innovational efforts of the GPT and the AS's unfold and interact over time is the theory of dynamic oligopoly as developed by Maskin and Tirole (1987) (henceforth M&T), which centers around the concept of Markov Perfect Equilibrium (MPE).[16] In what follows we sketch the model and (re)state the pertinent results from M&T in terms of GPT's and AS's.

Denote by $\pi^a(z_t, T_t)$ the instantaneous profit function of the AS and by $\pi^g(z_t, T_t)$ that of the GPT (for simplicity we assume that w is fixed).[17] The GPT and the AS are assumed to move in alternate periods of fixed length τ. In the present context, τ has a natural interpretation, namely, it is the length of time it takes to develop the 'next generation' (either of the GPT or of the AS), given that the other side has already developed its current technology. Thus, the quality level of the GPT at time $t-1$ is z_{t-1} and it remains constant for the next two periods (i.e., for a length of time of 2τ). Given z_{t-1}, the AS develops its technology up to level T_t, over a period of length τ. Similarly, after the realization of T_t it takes the GPT τ to develop *its* next generation, z_{t+1}, which will be marketed in period $t+1$. We refer throughout to a single AS facing the GPT, since the case with multiple AS's is far more complex and hard to analyze.[18]

With no adjustment costs, each firm maximizes at time t,

$$\sum_{s=0}^{\infty} \delta^S \pi^i(z_{t+s}, T_{t+s}), \qquad i = a, g,$$

where $\delta = \exp(-r\tau)$ is the discount factor and r is the interest rate. Define a dynamic reaction function for Markov strategies (i.e., dependent only on the payoff-relevant state) for the AS as $T_t = R^a(z_{t-1})$ and, similarly for the GPT,

[16] A more thorough treatment, incorporating uncertainty explicitly, would follow Pakes and McGuire (1992); however, that is well beyond the scope of this paper.

[17] We assume that these instantaneous profit functions have the same derivative properties as their static analogs of previous sections, but we further assume that $\pi^i(\cdot)$, $i = a, g$, are bounded from above.

[18] We conjecture that the qualitative results will be the same if instead there are a few large AS that act in tandem *vis-a-vis* the GPT or if the GPT acts as a Stackelberg leader *vis-a-vis* many small AS's; however, further work needs to be done to prove that this is so, in particular one would have to deal appropriately with the problem of multiple equilibria.

$z_t = R^g(T_{t-1})$. The pair (R^a, R^g) form a MPE iff there exist valuation functions (V^i, W^i), $i = a, g$, such that (for the AS)

$$V^a(z) = \max_T \,[\pi^a(z, T) + \delta W^a(T)],$$

$$R^a(z) \quad \text{maximizes} \quad [\pi^a(z, T) + \delta W^a(T)],$$

$$W^a(T) = \pi^a[R^g(T), T] + \delta\, V^a[R^g(T)],$$

and analogous conditions hold for the GPT's valuation functions. It is easy to show that the reaction functions will be upward-sloping in this case, since the cross-derivatives of the payoff functions, π^i_{zT}, are positive (because of innovational complementarities).[19]

M&T prove that, for any discount factor δ, (i) there exists a unique linear MPE which is dynamically stable, and (ii) the equilibrium (steady state) values of the decision variables (z^e, T^e in the present case) equal the static Cournot–Nash equilibrium when $\delta = 0$, and grow with δ. An equivalent way of phrasing (ii) is that the (dynamic) reaction functions coincide with their static (or 'Cournot') counterparts as δ goes to zero.[20]

In order to verify that this proposition holds also for the case of positively sloped reaction functions, we run simulations of the MPE that results from various values of the discount factor over its whole range (i.e., $\delta \in [0, 1]$). As shown in Appendix 2, the long-term equilibrium values $\{z^e, T^e\}$ increase indeed with the discount factor, and that is true for any value of the other parameter in the system.[21]

The dependence of the long-run equilibrium upon the discount rate has interesting implications in our context. In order to explore them we first modify the model to include 'adjustment costs' since it is not quite plausible that R&D costs will be a function of the *absolute* level of z (or T) that the firm wants to achieve. Rather, it is more likely that R&D costs depend upon the intended *increments* in technology, that is, that they are a function of $\Delta z_t = (z_t - z_{t-1})$, and similarly for T. M&T elaborate on the MPE that obtains in the case of

[19] See the proof of Lemma 1 in M&T (pp. 950–951): the negative slope of the reaction function stems directly from the assumption that $\pi_{12} < 0$. Thus, the converse holds for $\pi_{12} > 0$ (which is the equivalent of our $\pi_{zT} > 0$).

[20] M&T prove the proposition for the special case of quadratic profit functions; Dana and Montrucchio (1986) generalized the proof for any concave payoff function; see also Dana and Montrucchio (1987).

[21] The other parameter is d, the constant in the quadratic profit function, which enters multiplicatively in the equations for z^e and T^e, and hence does not affect the relationship between them and δ.

quadratic profit functions, when adjustment costs take the form $(\alpha/2)(z_t - z_{t-1})^2$, resulting in the linear dynamic reaction functions[22]

$$R^a(Z_{t-1}, T_{t-2}) = b_0 + b_1 z_{t-1} + b_2 T_{t-2},$$
$$R^g(z_{t-2}, T_{t-1}) = b_0 + b_1 T_{t-1} + b_2 z_{t-2}. \tag{9}$$

The long-term equilibrium values are then easily computed as $T^e = z^e = b_0/(1 - b_1 - b_2)$. Since even this simple case does not have closed form solutions (except in the limiting case of a large α), we resort once again to simulations and find that the discount factor plays here the same role as without adjustment costs, that is, the equilibrium values $\{z^e, T^e\}$ increase in δ (see Appendix 2).[23] Thus, the monotonicity of $\{z^e, T^e\}$ with respect to the discount factor generalizes both for the case of strategic complements, and for the case with adjustment costs.

In the current context the discount factor δ can be interpreted as a measure of the difficulty in forecasting the technological developments of the other side: the smaller δ is, the more difficult it is for the AS to anticipate the future quality of the GPT, and vice versa.[24] Technological forecasting, in turn, depends upon a variety of institutional arrangements that may facilitate or hinder the flow of credible technological information between the GPT and the AS's. Thus, the above results imply that the more 'cooperative' the GPT and the AS's are in terms of informational exchanges, the higher the ultimate equilibrium levels $\{z^e, T^e\}$ will be, and, since the reaction functions are positively sloped, the larger the values $\{z_t, T_t\}$ will be at each step in the sequence leading towards the steady state (see Fig. 2). Larger values at each step may translate in turn into faster aggregate growth, provided that in the process the GPT diffuses throughout a large number of sectors in the economy.

Recalling that $\delta = \exp(-r\tau)$, a useful way of thinking of δ in the present context is as follows: Suppose that τ is the required overall development time of

[22] To recall, since M&T assume that the cross-derivatives of the profit function are negative, b_1 is in their case negative. Keep in mind that $\{b_0, b_1, b_2\}$ are unknowns, that are obtained by solving the system for the MPE. Here resides the main practical difficulty of the model, since the system of equations that needs to be solved (by simulations) in order to obtain $\{b_0, b_1, b_2\}$ can be very complex.

[23] We also find that $\{z^e, T^e\}$ increase with the shift parameter of the profit function and decrease with α, but these are hardly surprising results. The simulations were run assuming symmetry between the GPT and the AS, which is in this case rather implausible (if only because there are no natural units to define z and T); that is, however, a mere technicality: the truly limiting assumption is the functional form of the profit and adjustment costs functions.

[24] This is of course a shortcut to the explicit modelling of technological uncertainty, which would involve a game of incomplete information.

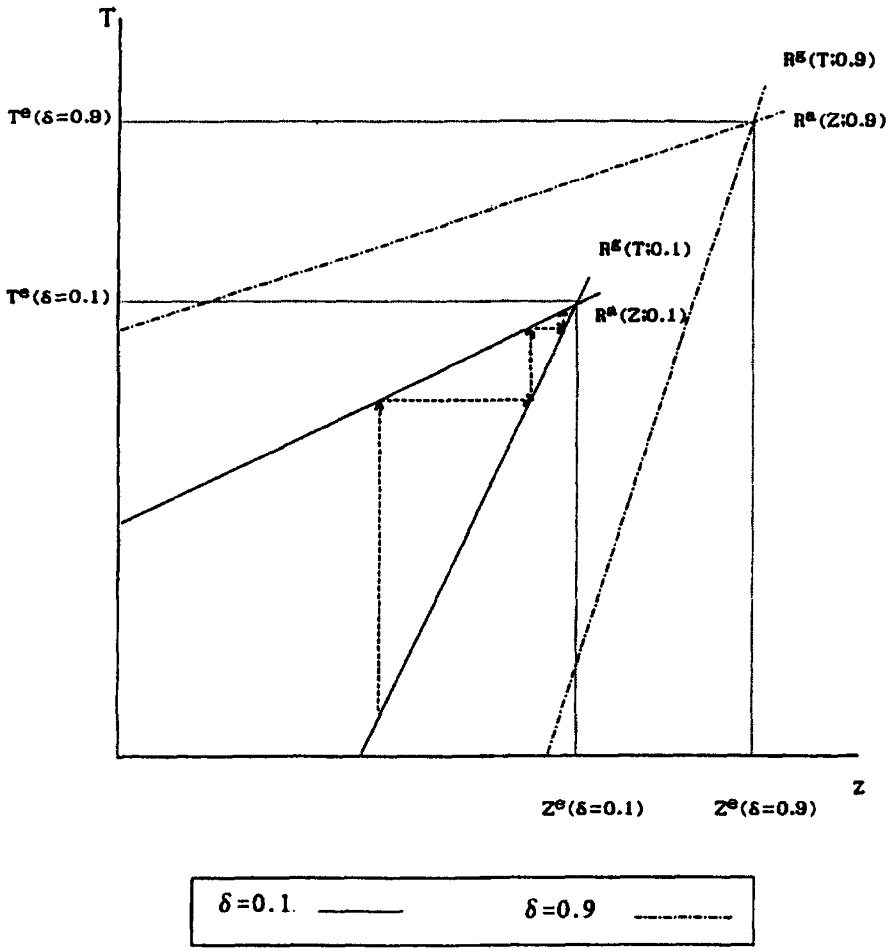


Fig. 2. Dynamic reaction functions, drawn on the basis of the numerical results shown in Table A.1, Case 1 [e.g., $z^e(\delta = 0.9) = T^e(\delta = 0.9) = 139$].

each 'new generation' of both the GPT and the AS. However, assume now that a proportion $(1 - \theta)$ of the development can be done before the other side has completed its development (which implies of course that a proportion θ *has to be done* afterwards). Thus, the 'effective' length of a period is $r^* \equiv \theta\tau$, $\theta \in [\underline{\theta}, \bar{\theta}]$, $\underline{\theta} > 0$, $\bar{\theta} \leqslant 1$; obviously, the smaller is θ, the larger δ will be [since $\delta = \exp(-\theta\tau^*) = \exp(-\theta\tau r)$].

If the relationship between the GPT and the AS takes the form of arms-length market transactions, with no intended exchange of technological information between them, then $\theta = \bar{\theta}$, hence δ will be small and so will $\{z^e, T^e\}$. On the other hand, if all technically relevant information flows freely between the two players, then $\theta = \underline{\theta}$, leading to faster innovation and higher levels of long-run equilibrium technologies. Thus the value of θ, reflecting institutional and organizational arrangements, may profoundly affect the present and future pace of innovation. Presumably, concerted action by the players involved as well as government policy may be able to alter θ and thus influence the rate of GPT-related technical change in the economy.

As an example, consider the case of Intel, Inc. *vis-a-vis* manufacturers of personal computers. The latter knew for quite a while that the next generation of Intel's microprocessors was the 586 (the 'Pentium'), that it was due in the spring of 1993, that it was expected to have at least twice the 486's performance (see Table 1), etc. On that basis they presumably were able to do part of the R&D for their next generation of PC's which will incorporate the 586. However, some of the development process requires that they actually get their hands on the 586,

Table 1
Successive generations of a GPT: Actual and expected

Intel's microprocessor dynasty

Chip	Introduced	
8086/8088	1978/1979	The chips that powered the first IBM PC's and PC clones; they crunch numbers in 16-bit chunks but have limitations in use of computer memory
80286	1982	Speedier than the 8088/8086, the 80286 also enabled computers to run for larger programs; first appeared on the 1984 IBM PC/AT
80386	1985	First Intel 32-bit microprocessor, capable of processing data in 32-bit chunks; gave PC's power to do bigger jobs, like running networks
80386SX	1988	Lower-priced version of the 80386, aimed at killing off the 80286, which was also produced by Advanced Micro Devices
80486	1989	Intel's 'mainframe on a chip'; with 1.2 million transistors, it is one of the most complex chips ever made
486SX	1991	The chip aimed at bringing mainframe power to the masses; it will eventually make the 80386 obsolete
586	1992	Expected to have 2 million transistors and at least twice the 80486's performance; its mission: to compete with RISC chips
686	1993/1994	Just entering the development phase, the 686 is likely to include sound and video-processing features for 'multimedia'

From *Business Week*, April 29, 1991, p. 55.

and test it in various configurations. How much they can develop prior to the actual appearance of the 586 depends *inter alia* upon the degree of detail of the technological information that they manage to obtain and the extent to which Intel is willing to make them privy of the development process.[25, 26]

On the other hand, coordination attempts can involve substantial informational and reputational costs which can make technology forecasting quite difficult, as revealed for example in the old dispute between IBM and manufacturers of competing mainframe system and 'plug-compatible' peripherals[27] or in the current complaints of software developers against Microsoft.[28]

Clearly, the scope for coordination in the sense outlined above increases with the number and range of AS's (and so does the loss in the case of a failure to coordinate). For example, an improvement in the ability of the PC industry to forecast technological advances in microprocessors may speed up the use of microelectronics in *cars*, fostering larger improvements in cars themselves, stimulating the demand for chips and encouraging their further development, and so forth.[29]

5. Concluding remarks

This paper focuses on the interface between 'key' technologies and the industrial organization of the markets and firms that spring up around them.

[25] It is interesting to note that, dramatically altering its conduct in this respect, Intel has been providing some of its users (such as Compaq) with details of the 586 as it was being developed.

[26] The reverse condition is perhaps less obvious but not less important: to continue with the same example, Intel has been developing parts and circuits for personal computers (other than microprocessors) because '··· through them Intel gains insight into trends: *Knowing what needs to go on a board this year helps it determine what should go into microprocessors next year*' (*Business Week*, April 29, 1991, p. 55). This is true to various degrees as one goes down the 'technological tree': thus, software developers need to actually have the new operating systems in order to develop software for them; in order to write new operating systems one needs to get one's hands on the (new) personal computers that will use them, and so forth.

[27] The latter accused IBM of attempting to delay their innovation efforts through concealment of information about interface standards and uncooperative behavior in establishing market-wide standards (e.g., ASCII vs. EBCDIC) (see Brock, 1975; Fisher et al., 1983).

[28] They claim that Microsoft is less than candid about the features of forthcoming operating systems, thereby delaying efforts to produce complementary applications. In these examples, struggles for market power may have lead to anti-coordination incentives, an idea familiar from the standards literature (see David and Greenstein, 1990). Farrell and Saloner (1986) offer a theory in which there is a social gain to coordinating but rent seeking behavior leads to imperfect outcomes.

[29] This has the flavor of the 'big push' in economic development (see Hirshman, 1960).

What makes them 'key' is their revealed dynamism and pervasiveness, which are endogenous to the system.[30] The goal is to forge a link between the incentives to innovate in GPT-AS's clusters and economic growth, which builds upon the industrial organization details of these markets. Our analysis shows that the unfolding of a GPT gives rise to increasing returns-to-scale, and that this plays an important role in determining the rate of technical advance in the cluster of associated sectors. On the other hand, this same phenomenon makes it difficult for a decentralized economy to fully exploit the growth opportunities offered by an evolving GPT. In particular, if the relationship between the GPT and its users is limited to arms-length market transactions there will be 'too little, too late' innovation in both the GPT and the application sectors. Likewise, difficulties in forecasting technological developments may lower the rate of technical advance of all sectors. Lastly, we have sketched a framework for the empirical analysis of GPT's as they interact with application sectors.

In future work we intend to follow several tracks. First, we would like to conduct empirical studies of GPT's as they evolve over time, interacting with a wide range of using sectors. The starting point would be the dynamic reaction functions in (9) (allowing for a multiplicity of AS's), which can be easily turned into a system of simultaneous equations having as endogenous variables $\dot{z}/z$ and $\dot{T}_a/T_a$ and as exogenous variables demand factors and the rate of advance of 'basic science' (i.e., advances that have a bearing on technical progress in the GPT, but that are not influenced themselves by the GPT). As empirical counterparts of $\dot{z}/z$ and $\dot{T}_a/T_a$ one could use a wide variety of patents measures, as suggested in Trajtenberg et al. (1992). Another possibility would be to use hedonic- based price indices as proxies for $\dot{z}/z$ and $\dot{T}_a/T_a$, but it is doubtful that one could obtain such indices for sufficiently many AS's. The key parameters of interest in such a system would be the slopes of the dynamic reaction functions, which determine the dynamic performance of the GPT-AS's cluster, and hence impact the growth of the whole economy.

Second, we would like to do micro-level studies, aimed at estimating 'technological value added': how much of the gains from innovation registered in markets for final products (i.e., the markets for the AS's) are 'due to' technological advances in the AS's themselves, as opposed to stemming from innovations in the GPT incorporated in the AS's? In our notation the issue is estimating and comparing π_z^a versus π_T^a. We have collected extensive data on microcomputers, which may allow us to carry out this type of study.

Third, we aim to carry out historical studies of GPT's and 'institutions' (in the broad sense): the intention would be to examine the historical evolution of particular GPT's and of the institutions coupled with them, using our

[30] Surely there are more primitive features that attest to the *potential* of some technologies to become GPT's, but so far we have not been able to find a convincing characterization of such features.

conceptual framework in trying to understand their joint dynamics. In particular, we would like to assess the extent to which specific institutions facilitated or hindered the GPT's in playing out their presumed roles as 'engines of growth'. A key hypothesis is that institutions display much more inertia than leading technologies. Thus, as a GPT era comes to a close and new GPT's emerge, an economy may 'get stuck' with the wrong institutions, that is, those that enable the previous GPT to advance and carry the AS's, but that may prove inadequate to do as much for the new GPT.

Appendix 1: Proof of upward-sloping $R^g(T)$

To show that $R^g(T)$ is upward-sloping, we perform the comparative statics exercise implied by maximizing Eq. (6) in the text, for a fixed A:

$$R^g_{T_a} \equiv \frac{\partial z}{\partial T_a} = \beta_1 X^a_{zT_a} + \beta_2 [X^a_{T_a} + (w-c) X^a_{wT}], \tag{10}$$

where

$$\beta_1 = \begin{Vmatrix} (w-c)\Sigma_A X^a_{zz} & \Sigma_A X^a_z + (w-c)\Sigma_A X^a_{zw} \\ \Sigma_A X^a + (w-c)\Sigma_A X^a_{zw} & 2\,\Sigma_A X^a_w + (w-c)\Sigma_A X^a_{ww} \end{Vmatrix}^{-1} \times (2\Sigma_A X^a_w + (w-c)\,\Sigma_A X^a_{ww})\,(c, w)$$

and

$$\beta_2 = \begin{Vmatrix} (w-c)\Sigma_A X^a_{ww} & \Sigma_A X^a_z + (w-c)\Sigma_A X^a_{zw} \\ \Sigma_A X^a + (w-c)\,\Sigma_A X^a_{zw} & 2\,\Sigma_A X^a_w + (w-c)\Sigma_A X^a_{ww} \end{Vmatrix}^{-1} \times (2\Sigma_A X^a_z + (w-c)\,\Sigma_A X^a_{zw}).$$

If the second-order conditions for a GPT profit maximum hold, our assumptions imply $\beta_1 > 0$, $\beta_2 > 0$, and hence $R^g_{T_a} > 0$. The intuition of this result is easy to see. High T_a in any AS shifts the demand for the GPT good out. The expectation would be, with innovational complementarities, that this raises the private return to investing in z. This argument is not quite complete, however. Since the GPT sector earns its private return through monopoly power, we need a further set of conditions that z and T_a shift marginal revenue in the same direction as the shift in demand – see numerator of β_2 and second term in Eq. (10).

To complete the proof, consider what happens when an additional AS enters. In the case of fixed w, it is immediate that an additional AS increases optimal z. When w is free to vary, the result is implied by our assumptions: add the marginal sector to Eq. (6) with weight λ and differentiate with respect to λ. At $\lambda = 0$, the impact on z is $\beta_1 X^a_z + \beta_2 [X^a + (w-c) X^a_w] > 0$. For $\lambda > 0$, the values of β_1 and β_2 change but are always positive. Thus, adding a sector always increases z.

Appendix 2: Simulations of MPE

Case 1: No adjustment costs

The profit functions are assumed to take the form:[31]

$$\pi_t^a = (d - T_t + z_t)T_t, \tag{11}$$

and similarly,

$$\pi_t^g = (d - z_t + T_t)z_t\,, \tag{12}$$

where d is a shift parameter common to both. Thus the reaction functions are

$$R^a(z_{t-1}) = b_0 + b_1 z_{t-1}\,,$$

$$R^g(T_{t-1}) = b_0 + b_1 T_{t-1}\,,$$

Solving for MPE renders the following two equations:[32]

$$b_1^4\delta^2 + 2\delta b_1^2 - 2b_1(1 + \delta) + 1 = 0, \tag{13}$$

$$b_0(1 - \delta b_1 \delta^2 b_1^2 - \delta^2 b_1^3) = db_1(1 + \delta). \tag{14}$$

Given δ one can solve for b_1 in (13), and then, given δ, d, and the corresponding b_1, one can solve for b_0 in (14). We solve for $[b_0, b_1]$ out of this system for different values of the parameter δ and compute the long-term equilibrium values,

$$T^e = z^e = \frac{b_0}{1 - b_1}\,.$$

As can be seen in (14), d impacts b_0 in a multiplicative fashion, but does not influence b_1; hence $\{z^e, T^e\}$ are just multiples of d, and we can perform all simulations with a single value of d (we picked $d = 100$). The results are shown in Table A.1.

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[31] We assume that d is large enough (relative to T and z) so that within the relevant range $\partial\pi^a/\partial T > 0$ and $\partial\pi^g/\partial z > 0$. For convenience we omit here the subindex a in the T's.

[32] Comparing this case (with $\pi_{zt} > 0$ and hence a positive b_1) to the one examined by M&T, one can see that our Eq. (13) is identical to their Eq. 20, but our Eq. (14) differs from their Eq. 21.

Table A.1
Simulations of MPE

Case 1: Without adjustment costs ($d = 100$)

δ	b	a	$z^e(\delta) = T^e(\delta)$
0.1	0.475	55.1	105
0.2	0.451	60.3	110
0.3	0.428	65.6	115
0.4	0.406	71.0	120
0.5	0.384	76.2	124
0.6	0.364	81.4	128
0.7	0.345	86.3	132
0.8	0.327	91.1	135
0.9	0.311	95.7	139

Case 2: With adjustment costs ($d = 100$)

	$z^e(\delta, \alpha) = T^e(\delta, \alpha)$		
δ	$\alpha = 1$	$\alpha = 10$	$\alpha = 100$
0.1	103.4	100.9	100.1
0.2	107.0	102.0	100.3
0.3	110.8	103.4	100.5
0.4	115.0	105.3	100.8
0.5	119.3	107.9	101.2
0.6	123.8	111.4	102.0
0.7	128.6	116.4	103.7
0.8	133.3	123.3	107.5
0.9	138.1	132.5	117.4

Case 2: With adjustment costs

The profit functions are the same as in (11) and (12), except that we subtract from them the adjustment costs $A^a = (\alpha/2)(T_t - T_{t-1})^2$ for the AS and $A^g = (\alpha/2)(z_t - z_{t-1})^2$ for the GPT sector. The corresponding reactions functions are

$$R^a(z_{t-1}, T_{t-2}) = b_0 + b_1 z_{t-1} + b_2 T_{t-2},$$

$$R^g(T_{t-1}, z_{t-2}) = b_0 + b_1 T_{t-1} + b_2 z_{t-2}.$$

The first-order condition is[33]

[33] Jean Tirole informed us in a personal communication that the FOC as shown in Eq. 30 of their published paper (Maskin and Tirole, 1987) is missing terms, and he kindly made available to me an unpublished corrigendum with the correct equation, which is the one shown here, adapted to our case.

$$\pi_z^g(z_t, T_{t-1}) - \frac{\partial A^g}{\partial z_t}(z_t, z_{t-2}) + \delta[\pi_z^g(z_t, T_{t+1}) + \pi_T^g(z_t, T_{t+1})]\frac{\partial R^a}{\partial z_t}(z_t, T_{t-1})$$

$$+ \delta^2[-\pi_z^g(z_{t+2}, T_{t+1})\beta + \pi_T^g(z_{t+2}, T_{t+1})]\frac{\partial R^a}{\partial z_t}(z_t, T_{t+1})$$

$$-\frac{\partial A^g}{\partial z_t}(z_{t+2}, z_t) + \beta\frac{\partial A^g}{\partial z_{t+2}}(z_{t+2}, z_t) + \delta^3[-\pi_z^g(z_{t+2}, T_{t+3})\beta]$$

$$+ \delta^4\left[\frac{\partial A^g}{\partial z_{t+2}}(z_{t+4}, z_{t+2})\beta\right] = 0.$$

Using the software program 'Mathematica' we derived from (15) the three equations in the three unknowns $[b_0, b_1, b_2]$, solve for them for different sets of values of the parameters $\{\delta, d, \alpha\}$, and compute the long-term equilibrium

$$T^e = z^e = \frac{b_0}{1 - b_1 - b_2}.$$

As in the case without adjustment costs, d impacts b_0 in a multiplicative fashion but does not influence b_1 and b_2; hence $\{z^e, T^e\}$ are multiples of d, and we can perform all simulations with a single value of d. Table A.1 shows the results of the simulations for $\alpha = 1, 10, 100$ and $\delta \varepsilon (0.1, 0.9)$ in intervals of 0.1.

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